Honors Algebra II Trigonometry **Name:** Key
2.4 – Linear Programming

1) A school board is investigating various ways of composing the faculty for a new elementary school. The school board can hire teachers and teacher aides. The board finds that the average teacher salary is $50,000 and the average teacher's aide salary is $45,000. The building can accommodate no more than 50 faculty members but must have a minimum of 20 faculty members to staff the school. There must be at least 12 teachers. Finally, the number of teachers must be at least half the number of aides. How many teachers and aides should the board hire to minimize cost?

a. Define the variables

X = # Teachers
Y = # Aides

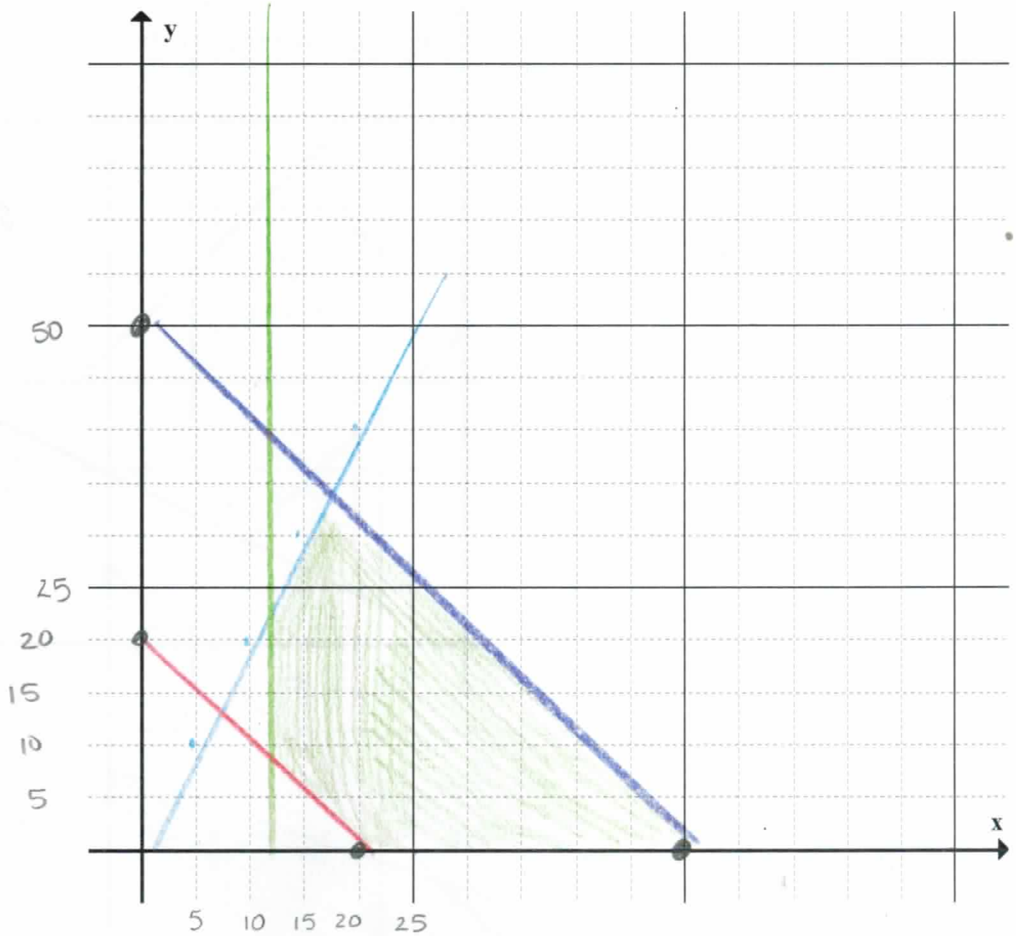
b. Constraints

$20 \le x + y \le 50$
$x \ge 12$
$x \ge \frac{1}{2}y$

c. Objective Function

$C = 50{,}000x + 45{,}000y$

d. Graph



(20,0) $1,000,000
(50,0) $2,500,000
(12,24) $1,680,000
$\boxed{(12, 8) \quad \$960{,}000}$
(16.7, 33.3) $2,333,500

$x + y = 50$
$x = \frac{1}{2}y$

$\frac{3}{2}y = 50$

$\boxed{y = 33.3 \quad x = 16.7}$

e. Minimize

We should hire 12 teachers and 8 aides

2) Monica Pety runs a gift shop where she sells expensive Christmas trees. Her supplier, Connie Furr, charges her $80 for each real tree and $160 for each artificial tree. She can buy between 20 and 90 real trees, and up to 100 artificial trees. Connie can supply anywhere between 60 and 120 total trees total, but requires that the number of artificial trees ordered be at least half the number of real trees. Monica wants to invest the minimum amount in trees. How many of each kind should she buy? What is her minimum feasible investment?

x = # real trees

y: # artificial trees

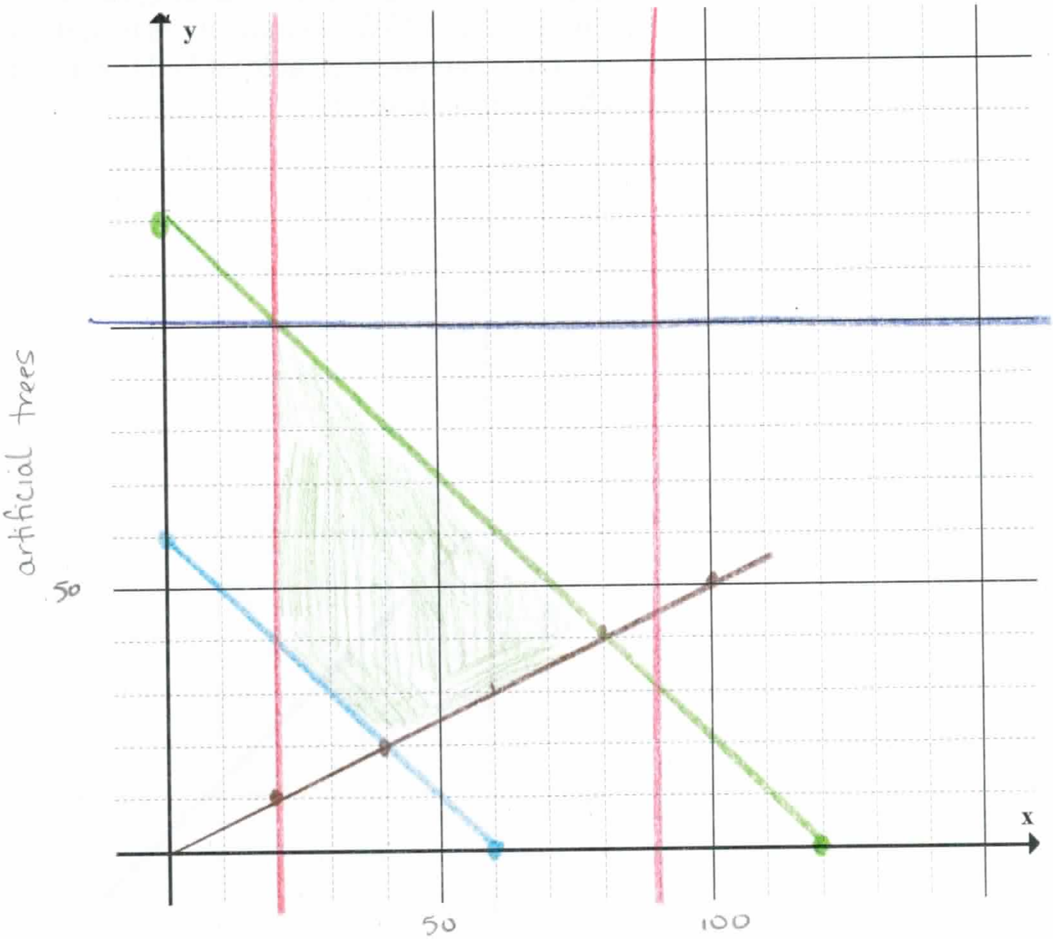
$20 \le x \le 90$

$y \le 100$

$60 \le x+y \le 120$

$y \ge \frac{1}{2}x$

$C = 80x + 160y$



(20,40) $8000

(40,20) $6400

(80,40) $12,800

(20,100) $17,600

She should buy 40 real trees and 20 artificial trees.
Her minimum investment is $6400.

3) Sabrina Burmeister is Chief Mathematician for Pedro Leum's Oil Refinery. Pedro can buy Texas oil, priced at \$30 per barrel, and California oil, priced at \$15 per barrel. He consults Sabrina to find out what is the most he might have to pay in a month for the oil that the refinery uses.

Sabrina finds the following restrictions on the amounts of oil that can be purchased in a month.

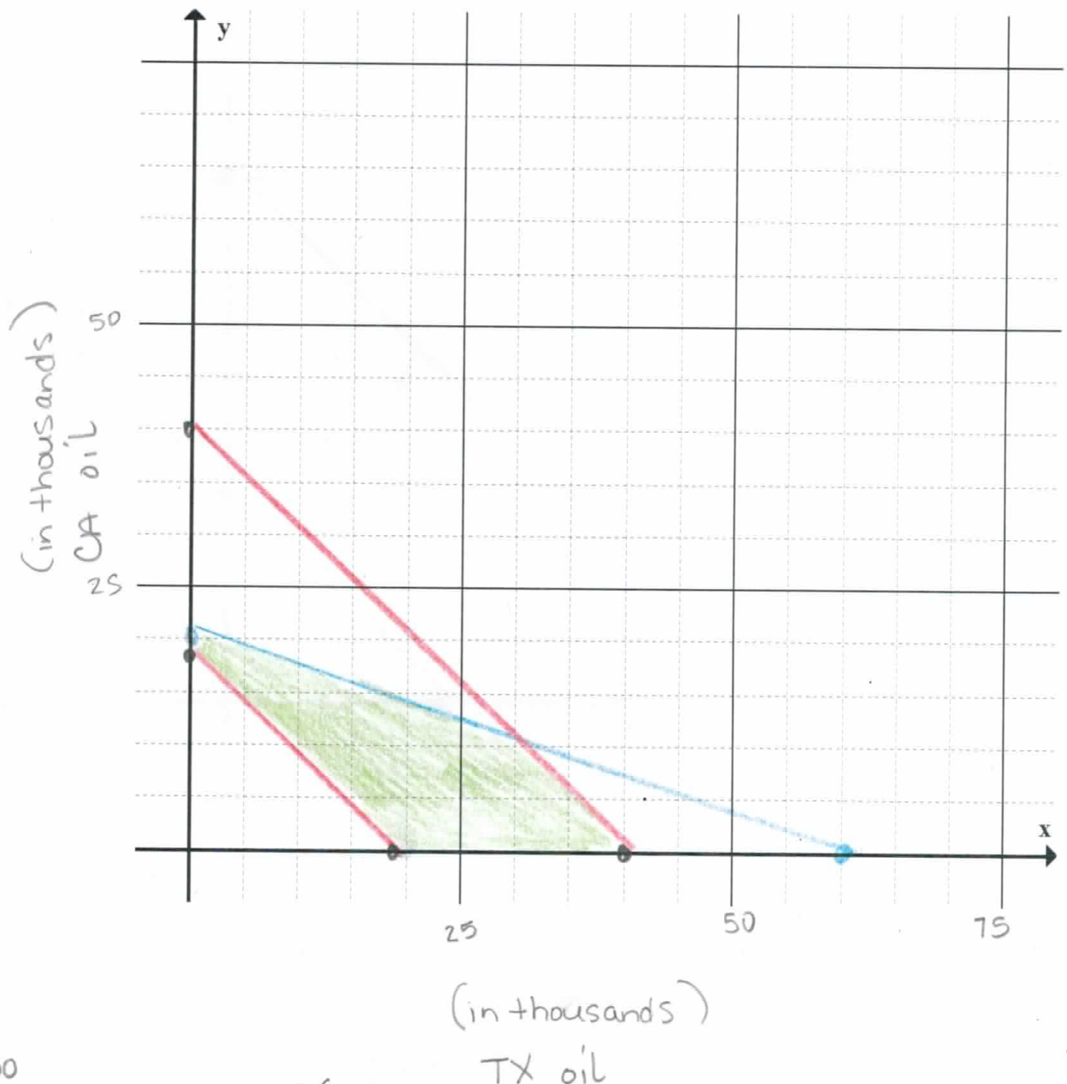
i. The refinery can handle as much as 40,000 barrels per month.
ii. To stay in business, the refinery must process at least 18,000 barrels a month.
iii. California oil has 6 pounds of impurities per barrel. Texas oil only has 2 pounds of impurities per barrel. The most the refinery can handle is 120,000 pounds of impurities a month.

What is the maximum feasible amount Pedro might have to spend in a month? How much of each kind of oil would give this maximum cost?

$C = 30x + 15y$

x = # TX barrels oil
y = # CA barrels oil

$x + y \leq 40{,}000$
$x + y \geq 18{,}000$
$2x + 6y \leq 120{,}000$



(0, 18,000) \$270,000
(0, 20,000) \$300,000
(18,000, 0) \$540,000
(40,000, 0) \$1,200,000
(30,000, 10,000) \$1,050,000

$-2(x + y = 40{,}000)$
$2x + 6y = 120{,}000$
$-2x - 2y = -80{,}000$
$4y = 40{,}000$
$y = 10{,}000$
$x = 30{,}000$

Max. feasible amt to spend is \$1,200,000 and 40,000 barrels of Texas oil

Solve each system. No calculator for these.

4) $\begin{cases} 3x - y = 5 \\ y = 4x + 2 \end{cases}$

$3x - (4x + 2) = 5$

$3x - 4x - 2 = 5$

$-x - 2 = 5$

$-x = 7$

$\boxed{x = -7}$

$y = 4(-7) + 2$

$\boxed{y = -26}$

5) $\begin{cases} 2x - y + z = -2 \\ x + 3y - z = 10 \\ x + 2z = -8 \end{cases}$

$3(2x - y + z = -2)$

$x + 3y - z = 10$

$6x - 3y + 3z = -6$

$7x + 2z = 4$

$7x + 2z = 4$

$-(x + 2z = -8)$

$6x = 12$

$\boxed{x = 2}$

$2 + 2z = -8$

$2z = -10$

$\boxed{z = -5}$

$2 + 3y + 5 = 10$

$3y + 7 = 10$

$3y = 3$

$\boxed{y = 1}$

6) $5\begin{cases} \frac{1}{x} + \frac{1}{y} = 8 \\ \frac{3}{x} - \frac{5}{y} = 0 \end{cases}$

$\frac{5}{x} + \frac{5}{y} = 40$

$\frac{8}{x} = 40$

$40x = 8$

$\boxed{x = \frac{1}{5}}$

$\frac{1}{1/5} + \frac{1}{y} = 8$

$5 + \frac{1}{y} = 8$

$\frac{1}{y} = 3$

$y = \frac{1}{\ }$