

Factor $f(x)$ into linear factors given that k is a zero of $f(x)$.

5. $f(x) = 2x^3 - 3x^2 - 5x + 6; k = 1$

$$\begin{array}{r|rrrr} & x^3 & x^2 & x & c \\ 1 & 2 & -3 & -5 & 6 \\ & \downarrow & & & \\ \hline & 2 & -1 & -6 & 0 \end{array}$$

$$\begin{array}{l} (x-1)(2x^2 - x - 6) \\ \boxed{(x-1)(2x+3)(x-2)} \end{array}$$

6. $f(x) = 6x^3 + 25x^2 + 3x - 4; k = -4$

$$\begin{array}{r|rrrr} -4 & x^3 & x^2 & x & c \\ & 6 & 25 & 3 & -4 \\ & \downarrow & & & \\ \hline & 6 & 1 & -1 & 0 \end{array}$$

$$\begin{array}{l} (x+4)(6x^2 + x - 1) \\ \boxed{(x+4)(3x-1)(2x+1)} \end{array}$$

For the polynomial function, one zero is given. Determine all others.

7. $f(x) = x^3 - x^2 - 4x - 6; x = 3$

$$\begin{array}{r|rrrr} & x^3 & x^2 & x & c \\ 3 & 1 & -1 & -4 & -6 \\ & \downarrow & & & \\ \hline & 1 & 2 & 2 & 0 \end{array}$$

Factors: $(x-3)(x^2 + 2x + 2)$

Zeros: $x = 3, -1 \pm i$

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 2}}{2} \\ &= \frac{-2 \pm \sqrt{-4}}{2} \\ &= \frac{-2 \pm 2i}{2} \\ &= -1 \pm i \end{aligned}$$

For each polynomial function, (a) list all possible rational zeros, (b) determine all rational zeros, and (c) factor $f(x)$.

1. $f(x) = x^3 + 3x^2 - 6x - 8$

(a) possible rational zeros: $\pm 1, 2, 4, 8$

(b) rational zeros: $x = -1, -4, 2$

(c) factor: $(x+1)(x+4)(x-2)$

$$\begin{array}{r|rrrr} & x^3 & x^2 & x & c \\ -1 & 1 & 3 & -6 & -8 \\ & \downarrow & -1 & -2 & 8 \\ \hline & 1 & 2 & -8 & 0 \end{array}$$

$x^2 + 2x - 8$
 $(x+4)(x-2)$

2. $f(x) = x^3 + 5x^2 + 2x - 8$

(a) possible rational zeros: $\pm 1, 2, 4, 8$

(b) rational zeros: $x = 1, -4, -2$

(c) factor: $(x-1)(x+4)(x+2)$

$$\begin{array}{r|rrrr} & x^3 & x^2 & x & c \\ 1 & 1 & 5 & 2 & -8 \\ & \downarrow & 1 & 6 & 8 \\ \hline & 1 & 6 & 8 & 0 \end{array}$$

$x^2 + 6x + 8$

3. $f(x) = x^3 + 9x^2 + 24x + 20$

(a) possible rational zeros: $\pm 1, 2, 4, 5, 10, 20$

(b) rational zeros: $x = -2, -5$

(c) factor: $(x+2)(x+5)(x+2)$

$$\begin{array}{r|rrrr} & x^3 & x^2 & x & c \\ -2 & 1 & 9 & 24 & 20 \\ & \downarrow & -2 & -14 & -20 \\ \hline & 1 & 7 & 10 & 0 \end{array}$$

$x^2 + 7x + 10$

4. $f(x) = x^3 - 2x^2 - 14x + 3$

(a) possible rational zeros: $\pm 1, 3$

(b) rational zeros: $x = -3$

(c) factor: $(x+3)(x^2 - 5x + 1)$

$$\begin{array}{r|rrrr} & x^3 & x^2 & x & c \\ -3 & 1 & -2 & -14 & 3 \\ & \downarrow & -3 & 15 & -3 \\ \hline & 1 & -5 & 1 & 0 \end{array}$$

$x^2 - 5x + 1$

other zeros are irrational