

Sketch the graph of the given function. Include the x-intercept(s), y-intercept, multiplicity of zeros, and end behavior.

1.  $f(x) = x^3 + 5x^2 - x - 5$

possible zeros:  $\pm 1, 5$

$$\begin{array}{r|rrrr} 1 & 1 & 5 & -1 & -5 \\ & \downarrow & & & \\ & 1 & 6 & 5 & 0 \end{array}$$

$(x-1)(x^2+6x+5)$

$f(x) = (x-1)(x+5)(x+1)$

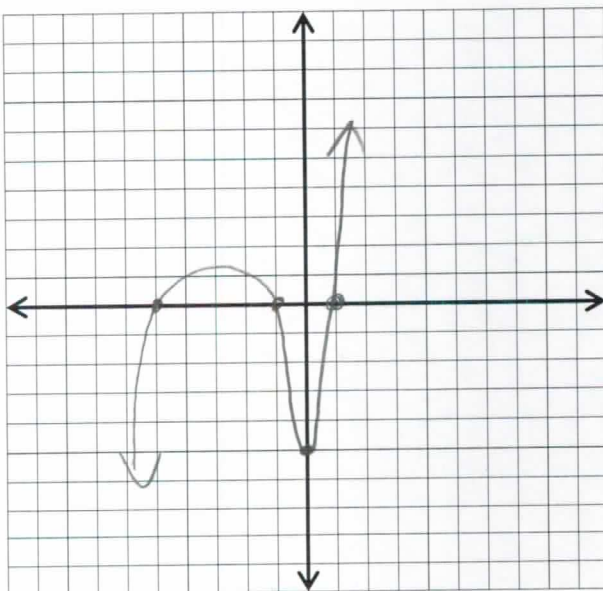
zeros:  $1, -5, -1$

y-int  $(0, -5)$

x-int  $(1, 0), (-5, 0), (-1, 0)$   
all cross

as  $x \rightarrow -\infty, f(x) \rightarrow -\infty$

as  $x \rightarrow \infty, f(x) \rightarrow \infty$



2.  $f(x) = x^3 + 5x^2 + 2x - 8$

possible zeros:  $\pm 1, 2, 4, 8$

$$\begin{array}{r|rrrr} 1 & 1 & 5 & 2 & -8 \\ & \downarrow & & & \\ & 1 & 6 & 8 & 0 \end{array}$$

$(x-1)(x^2+6x+8)$

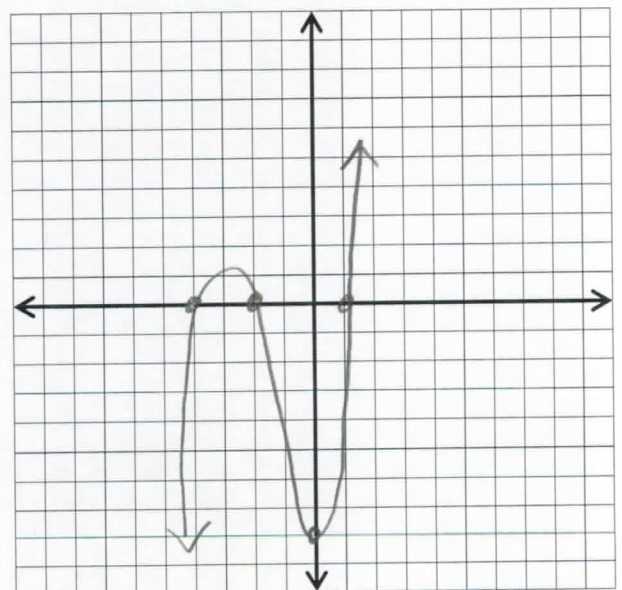
$f(x) = (x-1)(x+4)(x+2)$

y-int  $(0, -8)$

x-int  $(1, 0), (-4, 0), (-2, 0)$   
all cross

as  $x \rightarrow -\infty, f(x) \rightarrow -\infty$

as  $x \rightarrow \infty, f(x) \rightarrow \infty$



3.  $f(x) = x^4 + x^3 - 5x^2 + x - 6$

possible zeros:  $\pm 1, 2, 3, 6$

$$\begin{array}{r|rrrrr} 2 & 1 & 1 & -5 & 1 & -6 \\ & \downarrow & & & & \\ & 1 & 3 & 1 & 3 & 0 \end{array}$$

$$x^3 + 3x^2 + x + 3$$

$$x^2(x+3) + 1(x+3)$$

$$(x-2)(x+3)(x^2+1)$$

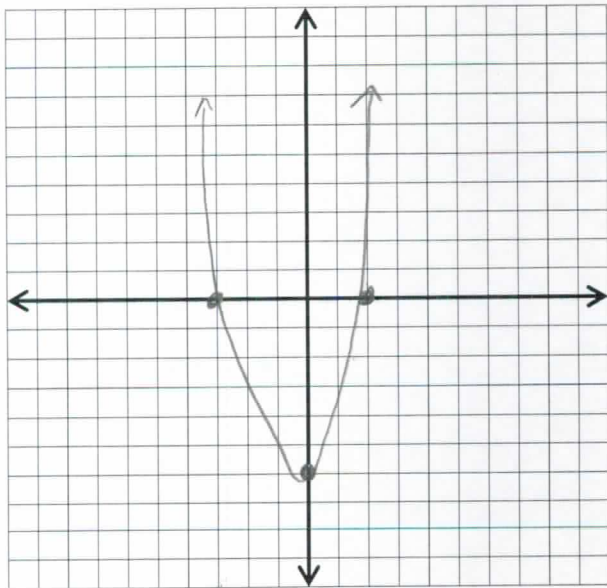
zeros:  $2, -3, \pm i$

y.int  $(0, -6)$

x.int  $(2, 0)(-3, 0)$   
cross cross

as  $x \rightarrow -\infty, f(x) \rightarrow \infty$

as  $x \rightarrow \infty, f(x) \rightarrow \infty$



4.  $f(x) = -x^4 - 3x^3 + 9x^2 + 23x + 12$

possible zeros:  $\pm 1, 2, 3, 4, 6, 12$

$$\begin{array}{r|rrrrr} -1 & -1 & -3 & 9 & 23 & 12 \\ & \downarrow & & & & \\ & -1 & -2 & 11 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & -1 & -2 & 11 & 12 & 0 \\ & \downarrow & & & & \\ & -1 & -1 & 12 & 0 & \end{array}$$

$$(x+1)^2(-x^2-x+12)$$

$$f(x) = -(x+1)^2(x+4)(x-3)$$

x.int  $(-1, 0)(-4, 0)(3, 0)$   
bounce cross cross

y.int  $(0, 12)$

as  $x \rightarrow -\infty, f(x) \rightarrow -\infty$

as  $x \rightarrow \infty, f(x) \rightarrow -\infty$

