

Determine the conic from the equation.

1. $x^2 + y^2 - 10x - 8y + 16 = 0$ circle
 $x^2 + 5y^2 - 10x - 8y + 16 = 0$ ellipse
 $x^2 - y^2 - 10x - 8y + 16 = 0$ hyperbola
 $x^2 - 10x - 8y + 16 = 0$ parabola

Write the equation of the conic described. Write the final answer in standard form.

2. For each point, its distance from the fixed point (4,3) is 3 times its distance from the fixed point (-1,2).

$$\sqrt{(x-4)^2 + (y-3)^2} = 3\sqrt{(x+1)^2 + (y-2)^2}$$

$$(x-4)^2 + (y-3)^2 = 9[(x+1)^2 + (y-2)^2]$$

$$x^2 - 8x + 16 + y^2 - 6y + 9 = 9(x^2 + 2x + 1 + y^2 - 4y + 4)$$

$$x^2 - 8x + y^2 - 6y + 25 = 9x^2 + 18x + 9y^2 - 36y + 45$$

$$0 = 8x^2 + 26x + 8y^2 - 30y + 20 \quad \text{or}$$

$$0 = 4x^2 + 13x + 4y^2 - 15y + 10$$

3. Each point is three times as far from the line $y=5$ as it is from the line $x=1$.

$$\sqrt{(x-x)^2 + (y-5)^2} = 3\sqrt{(x-1)^2 + (y-y)^2}$$

$$(y-5)^2 = 9[(x-1)^2 + (y-y)^2]$$

$$y^2 - 10y + 25 = 9(x^2 - 2x + 1)$$

$$y^2 - 10y + 25 = 9x^2 - 18x + 9$$

$$0 = 9x^2 - y^2 - 18x + 10y - 16$$

4. For each point, its distance from the point $(-3,1)$ is half its distance from the line $y=4$.

(x,y)

$$\sqrt{(x+3)^2 + (y-1)^2} = \frac{1}{2} \sqrt{(y-4)^2}$$
$$\left(2\sqrt{(x+3)^2 + (y-1)^2} \right)^2 = \left(\sqrt{(y-4)^2} \right)^2$$

$$4[(x^2 + 6x + 9 + y^2 - 2y + 1)] = y^2 - 8y + 16$$

$$4x^2 + 24x + 4y^2 - 8y + 40 = y^2 - 8y + 16$$

$$4x^2 + 24x + 3y^2 + 24 = 0$$

★ Rather than dealing with fractions, we can multiply both sides by 2.

5. For each point, its distance from the point $(4,0)$ is twice its distance from the line $x=-4$.

(x,y)

$$\left(\sqrt{(x-4)^2 + (y-0)^2} \right)^2 = \left(2\sqrt{(x+4)^2} \right)^2$$

$$(x-4)^2 + y^2 = 4(x+4)^2$$

$$x^2 - 8x + 16 + y^2 = 4(x^2 + 8x + 16)$$

$$x^2 - 8x + 16 + y^2 = 4x^2 + 32x + 64$$

$$0 = 3x^2 + 40x - y^2 + 48$$

6. Each point is equidistant from the point $(3,-4)$ and the line $y=2$.

(x,y)

$$\sqrt{(x-3)^2 + (y+4)^2} = \sqrt{(y-2)^2}$$

$$(x-3)^2 + (y+4)^2 = (y-2)^2$$

$$x^2 - 6x + 9 + y^2 + 8y + 16 = y^2 - 4y + 4$$

$$x^2 - 6x + 12y + 21 = 0$$