**Calculus**

**Ch 4 Part 2 – Day 4** Related Rates (Triangles & Circles)

Name: Key

Period: ________

Show all your work (diagram, equation relating variables, equation relating rates wrt time, substitution of information, answer with correct units):

1. A ladder 17ft long leans against a vertical building. If the bottom of the ladder slides away from the building horizontally at a rate of 3ft/sec, how fast is the ladder sliding down the building when the top of the ladder is 8ft from the ground?


$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

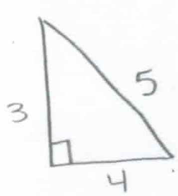
$$16\frac{dx}{dt} + 30\left(\frac{3ft}{sec}\right) = 0$$

$$16\frac{dx}{dt} = -30\frac{3ft}{sec}$$

$$\frac{dx}{dt} = -5.625\ ft/sec$$

The ladder is sliding down the wall at a rate of 5.625 ft/sec

2. A 5-ft ladder, leaning against a wall, slips so that its base moves away from the wall at a rate of 2 ft/sec. How fast will the top of the ladder be moving down the wall when the base is 4 ft from the wall?




$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

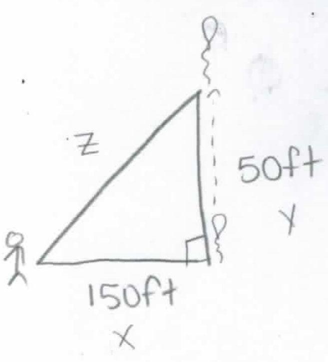
$$6\frac{dx}{dt} + 8\left(\frac{2ft}{sec}\right) = 0$$

$$6\frac{dx}{dt} = -\frac{16ft}{sec}$$

$$\frac{dx}{dt} = -2.67\ ft/sec$$

The ladder is sliding down the wall at a rate of 2.67 ft/sec

3. A small balloon is released at a point 150 feet away from an observer, who is on level ground. If the balloon goes straight up at a rate of 8 ft/s how fast is the distance from the observer to the balloon increasing when the balloon is 50 ft high?




$$50^2 + 150^2 = z^2$$

$$158.11 = z$$

$$x^2 + y^2 = z^2$$

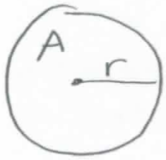
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$100ft\left(\frac{8ft}{sec}\right) = 316.22ft\frac{dz}{dt}$$

$$\frac{2.53ft}{sec} = \frac{dz}{dt}$$

The distance from the observer to the balloon is increasing at a rate of 2.53 ft/sec

4. An oil rig springs a leak in calm seas and the oil spreads in a circular patch around the rig. If the radius of the oil patch increases at a rate of 30m/hr, how fast is the area of the patch increasing when the patch has a radius of 100 meters?




$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{30m}{hr}$

$\frac{dA}{dt} = 2\pi(100m)\left(\frac{30m}{hr}\right)$

$= 6000\pi m^2/hr$

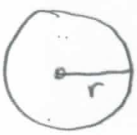
The area is increasing at a rate of $6000\pi\ m^2/hr$ $(18{,}849.6 m^2/hr)$

5. Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of 6 mi²/hr. How fast is the radius of the spill increasing when the area is 9 mi²?

$A = \pi r^2$

$9mi^2 = \pi \cdot r^2$

$1.69mi = r$




$A = \pi r^2$

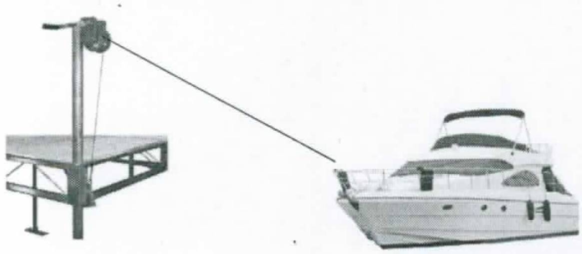
$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$\frac{6mi^2}{hr} = 2\pi(1.69 miles)\frac{dr}{dt}$

$\frac{dr}{dt} = .57mi/hr$

The radius is increasing at a rate of .57mi/hr

6. A boat is pulled into a dock by means of a rope attached to a pulley on the dock. The rope is attached to the bow of the boat at a point 10 ft below the pulley. If the rope is pulled through the pulley at a rate of 20 ft/min, at what rate will the boat be approaching the dock when 125 ft of rope is out?



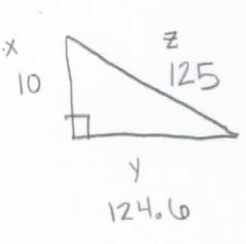
$\frac{dz}{dt} = -20ft/min$

Need to find $\frac{dy}{dt}$. X Never changes

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$0 + 249.2\frac{dy}{dt} = 250\left(\frac{-20ft}{min}\right)$




$\frac{dy}{dt} = -20.06 ft/min$

The boat is approaching the dock at a rate of 20.06ft/min.