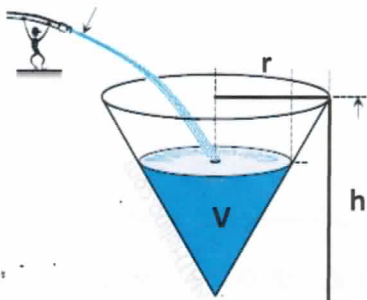


Calculus

Ch 4 Part 2 - Day 6 Notes and Worksheet Related Rates (Cones)

Name: Key

Write an equation that relates the following variables in each figure:



$$V = \frac{1}{3}\pi r^2 h$$

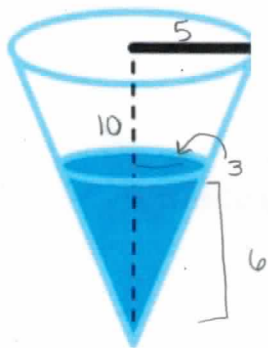
$$\frac{dV}{dt} = \frac{2}{3}\pi r h \frac{dr}{dt} + \frac{1}{3}\pi r^2 \frac{dh}{dt}$$

... if they don't give us 2 of the rates.

Note: While the product rule is used to find the derivative wrt time for the volume of a cone, substitution of the ratio of height to radius is needed when all necessary information is not provided.

Example #1: Filling a Conical Tank

Water runs into a conical tank at a rate of 9 ft³/min. The tank has a height of 10 ft and a radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?



$$V = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$$

$$V = \frac{1}{3}\pi \frac{1}{4}h^3$$

$$V = \frac{1}{12}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

$$9 = \frac{1}{4}\pi (6)^2 \frac{dh}{dt}$$

$$.318 \text{ ft/min} = \frac{dh}{dt}$$

$$\frac{r}{h} = \frac{1}{2}$$

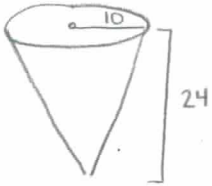
$$r = \frac{1}{2}h$$

Solve for $\frac{dh}{dt}$

The water level rises at a rate of .318 ft/min

Show all your work (diagram, equation relating variables, equation relating rates wrt time, substitution of information, answer with correct units) and write your final answer as a sentence:

1.) A conical water tank with vertex down has a radius of 10 ft at the top and is 24 ft high. If water flows out of the tank at a rate of $20 \text{ ft}^3/\text{min}$, how fast is the depth of the water decreasing when the water is 16 ft deep?



$$V = \frac{1}{3} \pi \left(\frac{5}{12} h \right)^2 h$$

$$V = \frac{25\pi}{432} h^3$$

$$\frac{dV}{dt} = \frac{75\pi}{432} h^2 \cdot \frac{dh}{dt}$$

$$-20 = \frac{75\pi}{432} (16)^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = -20 \text{ ft}^3/\text{min}$$

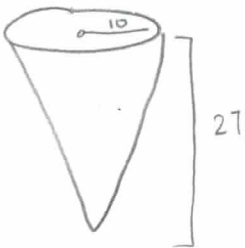
$$\frac{r}{h} = \frac{10}{24}$$

$$r = \frac{5}{12} h$$

$$-.143 \frac{\text{ft}}{\text{min}} = \frac{dh}{dt}$$

The water level is decreasing at a rate of $.143 \text{ ft}/\text{min}$

2.) A conical water tank with vertex down has a radius of 10 feet at the top and is 27 feet high. If water flows into the tank at a rate of $15 \text{ ft}^3/\text{min}$, how fast is the depth of the water increasing when the water is 18 feet deep?



$$V = \frac{1}{3} \pi \left(\frac{10}{27} h \right)^2 h$$

$$V = \frac{100\pi}{2187} h^3$$

$$\frac{dV}{dt} = \frac{100\pi}{729} h^2 \frac{dh}{dt}$$

$$15 = \frac{100\pi}{729} (18)^2 \frac{dh}{dt}$$

$$\frac{r}{h} = \frac{10}{27}$$

$$r = \frac{10}{27} h$$

$$.107 \frac{\text{ft}}{\text{min}} = \frac{dh}{dt}$$

The water level is increasing at a rate of $.107 \text{ ft}/\text{min}$