Calculus
Chapter 4, Part 2 Review

Name Key
Date________ **Per**______

Test Topic	Ready!
I can find the derivative of implicitly defined functions.	
I can find the equation of a tangent and normal line of implicitly defined functions	
I can use substitution to find the second derivative of implicitly defined functions.	
I can determine the solution strategy for related rates problems: • Pythagorean Theorem • Area of Circle • Area of Triangle • Right Triangle Trig (θ) • Volume/Area of Sphere • Volume of Cylinder • Volume of Cone • Volume and Surface Area of Cube	
I understand how the sign of a rate relates to increasing and decreasing.	

1.) Find the equation of the lines that are tangent and normal to the curve $xy + y^2 = 5$ at the given point $(1, 2)$.

$$x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x+2y) = -y$$

$$\frac{dy}{dx} = \frac{-y}{x+2y}$$

$$\frac{dy}{dx} = \frac{-2}{1+4} = \frac{-2}{5}$$

Tangent $y - 2 = \frac{-2}{5}(x-1)$

Normal $y - 2 = \frac{5}{2}(x-1)$

2.) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. $x^2 - y^2 = 3$

$$2x - 2y\frac{dy}{dx} = 0$$

$$-2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y - x\frac{dy}{dx}}{y^2}$$

$$= \frac{y - x \cdot \frac{x}{y}}{y^2} \cdot \frac{y}{y}$$

$$= \frac{y^2 - x^2}{y^3} = \frac{-1(x^2-y^2)}{y^3} = \frac{-1(3)}{y^3}$$

$\frac{dy}{dx} =$ $\frac{x}{y}$

$\frac{d^2y}{dx^2} =$ $\frac{-3}{y^3}$

Draw a picture and show all work.

3.) A metal disk expands during heating. If its diameter increases at the rate of 0.3 in/s, how fast is the area of increasing when its radius is 8.1 in?

$\frac{dd}{dt} = .3 \text{ in/sec}$

$d = 16.2$

$A = \pi\left(\frac{1}{2}d\right)^2$

$= \frac{1}{4}\pi d^2$

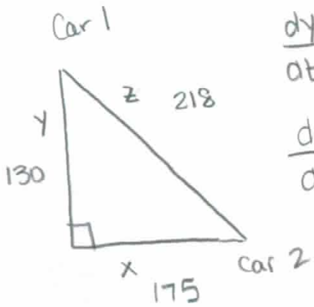
$\frac{dA}{dt} = \frac{1}{2}\pi d \cdot \frac{dd}{dt}$

$\frac{dA}{dt} = \frac{1}{2}\pi(16.2)(.3)$

$= 7.63 \text{ in}^2/\text{sec}$

The area increases at a rate of $7.63 \text{ in}^2/\text{sec}$

4.) Two cars are **approaching** an intersection along roads that make a 90° angle. The first car is 130 meters from the intersection and is traveling at a rate of 45 m/sec. The second car is 175 m from the intersection and is traveling at a rate of 25 m/s. Find the rate of change of the distance between the cars.



$\frac{dy}{dt} = -45 \text{ m/sec}$

$\frac{dx}{dt} = -25 \text{ m/sec}$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(175)(-25) + 2(130)(-45) = 2(218)\frac{dz}{dt}$

$\boxed{\frac{dz}{dt} = -46.9 \text{ m/sec}}$

5.) Sand pouring from a chute forms a conical pile whose height is always equal to the diameter. If the radius increases at a constant rate of 5 ft/min, at what rate is sand pouring from the chute at the instant the diameter of the base is 20 ft. wide?

$\frac{dr}{dt} = 5 \text{ ft/min}$

$r = \frac{1}{2}h$

$h = 2r$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2(2r)$

$= \frac{2}{3}\pi r^3$

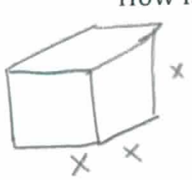
$\frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 2\pi(10)^2 \cdot 5$

$= 1000\pi \text{ ft}^3/\text{min}$

$\frac{dV}{dt} = 3141.59 \text{ ft}^3/\text{min}$

6.) At a certain instant each edge of a cube is 5 in. long and the volume is increasing at the rate of 2 in³/min. How fast is the surface area of the cube increasing?



$V = x^3$

$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$

$2 = 3(5)^2\frac{dx}{dt}$

$\frac{dx}{dt} = .027 \text{ in/min}$

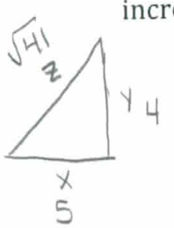
$SA = 6x^2$

$\frac{dSA}{dt} = 12x\frac{dx}{dt}$

$\frac{dSA}{dt} = 12(5)(.027)$

$\boxed{\frac{dSA}{dt} = 1.6 \text{ in}^2/\text{min}}$

7.) A rocket, rising vertically, is tracked by a radar station that is on the ground 5 mi from the Launchpad. How fast is the rocket rising when it is 4 mi high and its distance from the radar station is increasing at a rate of 2000 mi/hr?



$\frac{dz}{dt} = 2000 \text{ mi/hr}$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(4)\frac{dy}{dt} = 2\sqrt{41}(2000)$

$\boxed{\frac{dy}{dt} = 3201.56 \text{ mi/hr}}$