

Solve each equation. No calculator.

$$1. \quad 2^{3x} = 8\sqrt{2}$$

$$2^{3x} = 2^3 \cdot 2^{1/2}$$

$$2^{3x} = 2^{7/2}$$

$$3x = \frac{7}{2}$$

$$x = 7/6$$

$$2. \quad 3^{4x} - 3 = 24$$

$$3^{4x} = 27$$

$$3^{4x} = 3^3$$

$$4x = 3$$

$$x = 3/4$$

$$3. \quad \log_{10}(x^2 + 21x) = 2$$

$$10^2 = x^2 + 21x$$

$$0 = x^2 + 21x - 100$$

$$0 = (x+25)(x-4)$$

$$x = -25, 4$$

$$4. \quad 3^{2x} + 7 = 10$$

$$3^{2x} = 3$$

$$2x = 1$$

$$x = 1/2$$

$$5. \quad \log_8 \sqrt[6]{4^5} = x$$

$$8^x = \sqrt[6]{4^5}$$

$$2^{3x} = (2^2)^{5/6}$$

$$2^{3x} = 2^{5/3}$$

$$3x = \frac{5}{3}$$

$$x = 5/9$$

$$6. \quad \log_2(x^2 + 2x) = \log_2 8$$

$$x^2 + 2x = 8$$

$$x^2 + 2x - 8 = 0$$

$$(x+4)(x-2) = 0$$

$$x = -4, 2$$

Solve each equation. Calculator permitted.

$$7. \quad 2e^{2x} + 3 = 8$$

$$2e^{2x} = 5$$

$$e^{2x} = \frac{5}{2}$$

$$\ln \frac{5}{2} = 2x$$

$$.458 = x$$

$$8. \quad 3^{4x} - 3 = 4$$

$$3^{4x} = 7$$

$$\log_3 7 = 4x$$

$$.443 = x$$

$$9. \quad \ln(3x+1) - 4 = -2$$

$$\ln(3x+1) = 2$$

$$e^2 = 3x+1$$

$$2.13 = x$$

Simplify. No calculator.

$$10. \quad \sqrt[3]{16} \div \sqrt[4]{32}$$

$$\frac{2^{4/3}}{2^{5/4}}$$

$$2^{1/2}$$

$$11. \quad \frac{32^{1.4}}{16^{1.5}}$$

$$\frac{(2^5)^{1.4}}{(2^4)^{1.5}}$$

$$\frac{2^7}{2^6}$$

$$2$$

$$12. \quad \sqrt[3]{\sqrt{27}}$$

$$(3^3)^{1/2})^{1/3}$$

$$3^{1/2}$$

$$\frac{4}{3} - \frac{5}{4}$$

$$\frac{16}{12} - \frac{15}{12}$$

$$8^x = 16 \quad 3^x = 27$$

$$2^{3x} = 2^4 \quad x = \frac{4}{3}$$

13. $\frac{\sqrt[3]{128}}{\sqrt{32}}$

$$\frac{(2^7)^{\frac{1}{3}}}{(2^5)^{\frac{1}{2}}} = \frac{2^{7/3}}{2^{5/2}} = 2^{-1/6}$$

$$\frac{7}{3} - \frac{5}{2} = \frac{14}{6} - \frac{15}{6} = -\frac{1}{6}$$

14. $\log_{16} 4\sqrt{2} = x$

$$16^x = 4\sqrt{2}$$

$$2^{4x} = 2^2 \cdot 2^{1/2}$$

$$2^{4x} = 2^{5/2}$$

$$4x = \frac{5}{2}$$

$$x = \frac{5}{8}$$

15. $3\log_3 \frac{1}{27} + \frac{1}{4} \log_8 16$

$$3(-3) + \frac{1}{4} \left(\frac{4}{3} \right)$$

$$-9 + \frac{1}{3}$$

$$\frac{-26}{3}$$

16. Find the inverse of $f(x) = \log_2(x+3) - 1$.

$$x = \log_2(y+3) - 1$$

$$x+1 = \log_2(y+3)$$

$$2^{x+1} = y+3$$

$$2^{x+1} - 3 = y$$

$$f^{-1}(x) = 2^{x+1} - 3$$

Calculator permitted.

17. During the first stages of an epidemic, the number of sick people increases exponentially with time. Suppose that at time $t=0$ days, there are 40 sick people. By the time $t=3$, 200 people are sick. How many people will be sick by the time $t=6$? When will the number of sick people reach 7000?

$$y = a \cdot b^x$$

$$y = 40(1.70997)^x$$

$$y = 40(1.70997)^6$$

$$1000 \text{ people are sick when } t=6$$

$$7000 = 40(1.70997)^x$$

$$175 = 1.70997^x$$

$$\log_{1.70997} 175 = x$$

$$x = 9.63 \text{ days}$$

18. You invest \$350 at 6% compounded monthly. What is your balance after 5 years? How long does it take to double your investment?

$$y = 350 \left(1 + \frac{.06}{12} \right)^{12 \cdot 5}$$

$$= \$472.10$$

$$700 = 350 \left(1 + \frac{.06}{12} \right)^{12t}$$

$$2 = \left(1 + \frac{.06}{12} \right)^{12t}$$

$$\log_{1.005} 2 = 12t$$

$$11.58 \text{ yrs} = t$$